

Rotational and Reflectional Symmetry in Escher's Sketches

Objective: Look at symmetry in larger decorations. Learn to recognize rotations and reflections in wallpaper patterns.

The plane filling patterns (also called wallpaper patterns) shown on pages 116 - 233 of *Visions of Symmetry* exhibit rotational and/or reflectional symmetries. We have not really defined what a wallpaper pattern is at this point, but we do not need to know much about them in order to investigate the symmetries present in these patterns. The patterns shown are only part of the infinite pattern. In other words, we need to think of these patterns as extending infinitely far in all directions. We will return to the concept of wallpaper patterns later in the course.

Example 1 Look at Sketch #3 (Weightlifter) on page 117. There is 2-fold rotational symmetry where the heads meet. There is 4-fold rotational symmetry where the elbows meet. There are lines of rotational reflection running through the centres of the bodies.

Example 2 Look at Sketch #5 (Strongmen) on page 119. There are no lines of reflection for this pattern. We do find rotational symmetry. Note that there is 2-fold rotational symmetry. Also note that there are two different rotational symmetries at work here. There is a 2-fold rotation around a point on the hand of the figure, and there is a 2-fold rotation about a point on the upper arm of the figure. These are considered different centers of rotation!

Checklist

Sketch	Reflectional Symmetry (Yes or No)	Rotational Symmetry (2-, 3-, 4- or 6-fold; Where?)
Sketch #3 (Weightlifter) (pg 117)	Yes	2-fold at the head; 4-fold at the elbows
Sketch #4 (Chinese boy) (pg 118)		
Sketch #5 (Strongman) (pg 119)	No	2-fold at hands and left arms
Sketch #12 (Butterfly) (pg 125)		
Sketch #13 (Dragonfly) (pg 126)		

Sketch #25 (Lizard) (pg 135)		
Sketch #35 (Lizard) (pg 141)		
Sketch #45 (Angel-Devil) (pg 150)		
Sketch #85 (Lizard/Fish/Bat) (pg 184)		
Sketch #91 (Beetles) (pg 190)		

Additional Questions

1. Sketch #70 (Butterflies) has 6-fold rotational symmetry. Find at least two more numbered sketches with 6-fold rotational symmetry.

2. In some of the sketches we can see the underlying geometric pattern that Escher used to create these tessellations. It's easy to see for instance that in Sketch #3 (Weightlifter) there is an underlying tessellation by squares and in Sketch #4 (Chinese boy) there is an underlying tessellation by triangles. Where are the centers of rotation with respect to the underlying geometric tessellations? I.e. do they lie on the vertices? In the middle of the polygon? On the edges? Is there any pattern you can discern?

Hand in: The checklist; Answers to the additional questions